# Polarization and the Electromagnetic Spectrum

PHYS 2203 — Introductory Physics III

August 28, 2026

### Overview

This lecture continues our study of light. We first complete the mechanical properties of light by finding the force and pressure a light wave exerts on a surface, the radiation pressure. We then introduce the *polarization* of light — the direction in which $\vec{E}$ oscillates — and how polarizers act on it, and we survey the electromagnetic spectrum.

### Recap: Intensity and Momentum of a Wave

Last lecture we described a plane electromagnetic wave in vacuum, $\vec{E} = E_0 \sin(kx - \omega t)\,\hat{y}$ and $\vec{B} = B_0 \sin(kx - \omega t)\,\hat{z}$, with $\vec{E}$ and $\vec{B}$ in phase, mutually perpendicular, and related by $E_0 = cB_0$. Two results carry into this lecture.

- The **intensity** is the time-averaged power per unit area, $I = \frac{1}{2} c\varepsilon_0 E_0^2$, with units W/m$^2$.
- The wave carries **momentum** in proportion to its energy, $p = U/c$, a property of the wave itself.

We use both results immediately, starting with the pressure light exerts on a surface.

## 1 Radiation Pressure

Because a light wave carries momentum, it exerts a force on any surface that absorbs or reflects it.

Consider a plane wave incident on a surface of area $A$ over a time interval $\Delta t$. Let $\Delta U$ be the energy incident on the surface during this interval. To express $\Delta U$ through the intensity, note that the incident energy accumulates at the instantaneous rate $dU/dt$, the incident power. Its time average over $\Delta t$, evaluated by the fundamental theorem of calculus with $U(0) = 0$ and $U(\Delta t) = \Delta U$, is

$$\left\langle \frac{dU}{dt} \right\rangle_t = \frac{1}{\Delta t} \int_0^{\Delta t} \frac{dU}{dt}\, dt = \frac{U(\Delta t) - U(0)}{\Delta t} = \frac{\Delta U}{\Delta t}. \tag{1}$$

Intensity is the incident power per unit area, so $\Delta U/\Delta t = IA$ and $\Delta U = IA\,\Delta t$.

By the result found above, this incident light carries momentum $p = \Delta U/c$ toward the surface. First suppose the surface completely absorbs the wave. It takes in all of this momentum, so it starts at rest and ends with the light's momentum:

$$\Delta p = p_{\text{final}} - p_{\text{initial}} = \frac{\Delta U}{c} - 0 = \frac{\Delta U}{c}. \tag{2}$$

The force is the rate of momentum transfer,

$$\boxed{F \;=\; \frac{\Delta p}{\Delta t} \;=\; \frac{\Delta U}{c\,\Delta t} \;=\; \frac{IA}{c}.} \tag{3}$$

The corresponding pressure $P = F/A = I/c$ is the **radiation pressure**.

Now suppose the surface is a perfect reflector. It absorbs no energy, so it returns the full incident energy $\Delta U$ as reflected light, which by $p = U/c$ carries the same momentum magnitude reversed in direction, from $+\Delta U/c$ to $-\Delta U/c$. The surface gains twice as much,

$$\Delta p \;=\; \frac{\Delta U}{c} - \left(-\frac{\Delta U}{c}\right) \;=\; \frac{2\,\Delta U}{c}, \tag{4}$$

and the force and pressure double to $F = 2IA/c$ and $P = 2I/c$. (A ball delivers momentum $mv$ to a wall if it sticks, but $2mv$ if it bounces straight back.)

## 2 Polarization

Beyond its frequency, wavelength, and intensity, an electromagnetic wave possesses an additional structural property: the direction in which $\vec{E}$ oscillates within the plane transverse to propagation. This direction is called the **polarization** of the wave.

Consider a wave propagating along $+\hat{x}$ with electric field

$$\vec{E}(x,t) \;=\; E_0 \,\sin(kx - \omega t)\,\hat{y}. \tag{5}$$

The electric field at every position oscillates along $\hat{y}$, and the magnetic field oscillates along $\hat{z}$ (as required by $\hat{E} \times \hat{B} = \hat{x}$, the propagation direction). The $x$–$y$ plane is the *plane of polarization* of the wave. Such a wave is said to be **plane-polarized** (or linearly polarized) in the $\hat{y}$ direction.

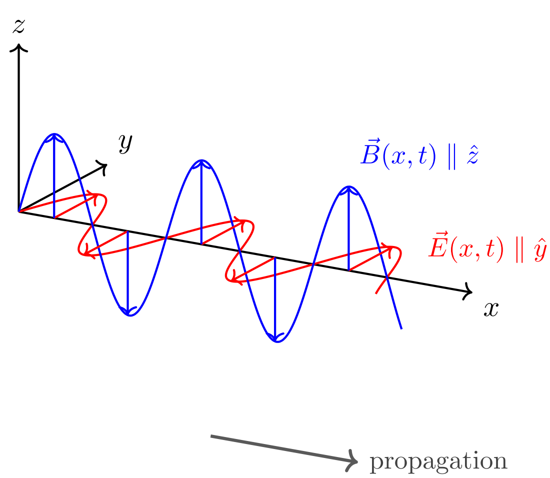


Figure 1: Plane-polarized electromagnetic wave traveling in $+\hat{x}$ with $\vec{E}$ oscillating along $\hat{y}$ (red) and $\vec{B}$ along $\hat{z}$ (blue). The two fields are in phase and mutually perpendicular. The triad $(\hat{x}, \hat{y}, \hat{z})$ is right-handed, and the direction of propagation is $\hat{x} = \hat{E} \times \hat{B} = \hat{y} \times \hat{z}$. The $x$–$y$ plane is the plane of polarization.

**Definitions**

- **Polarized:** For all positions and times the electric field (and therefore the magnetic field) oscillates along a definite direction or follows a definite pattern in the transverse plane.

- **Plane-polarized (linearly polarized):** The plane of polarization is independent of time. The electric field oscillates along a fixed direction in the transverse plane.

- **Circularly polarized:** The plane of polarization rotates about the propagation axis at angular rate $\omega$, while the amplitude $E_0$ remains constant. Mathematically, the two transverse components have equal amplitude but oscillate a quarter cycle out of phase:

$$\vec{E}(x,t) \;=\; E_0\big[\sin(kx-\omega t)\,\hat{y} \;+\; \cos(kx-\omega t)\,\hat{z}\big]. \tag{6}$$

  Since $\sin^2+\cos^2 = 1$, the tip of $\vec{E}$ at any fixed point traces a circle of radius $E_0$ in the transverse plane as time advances.

- **Unpolarized:** The direction of $\vec{E}$ in the transverse plane is random and varies on short time scales. Natural sources such as incandescent bulbs and the Sun emit unpolarized light. An unpolarized beam can be modeled as an incoherent superposition of two perpendicular linearly polarized beams: half the intensity with $\vec{E} \parallel \hat{y}$ and half with $\vec{E} \parallel \hat{z}$.

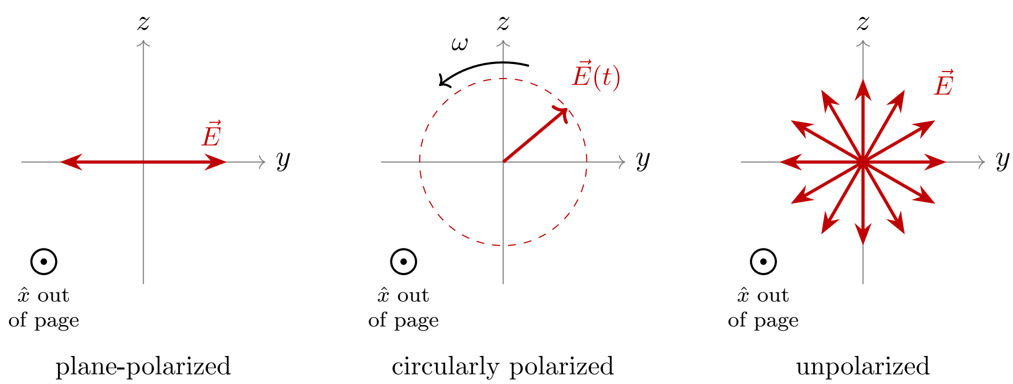


Figure 2: Head-on view of the transverse plane, with propagation $\hat{x}$ out of the page. Left: plane-polarized light. $\vec{E}$ oscillates along one fixed transverse direction (here $\hat{y}$). Middle: circularly polarized light. $\vec{E}$ has constant magnitude $E_0$, and its tip traces a circle (dashed) at angular rate $\omega$. Right: unpolarized light. The direction of $\vec{E}$ changes randomly and rapidly in time, and the arrows are a time-exposure of many oscillation directions, none favored on average.

## 3 Polarizers and Malus's Law

A **polarizer** is an optical element that transmits only the component of $\vec{E}$ along a particular axis (the polarizer's transmission axis) and absorbs the perpendicular component. The transmitted wave is plane-polarized along the transmission axis, regardless of the polarization state of the incident wave.

**Unpolarized light incident on a polarizer**

For an unpolarized beam of intensity $I_0$ incident on a polarizer, the transmitted intensity is

$$I_1 \;=\; \frac{1}{2}\,I_0, \tag{7}$$

because half the average electric-field energy is contained in each of the two orthogonal polarization components in the unpolarized superposition.

**Plane-polarized light incident on a polarizer: vector decomposition**

If the light leaving the first polarizer is polarized along $\hat{y}$, and the second polarizer's axis is at angle $\theta$ from $\hat{y}$, how does *any* light get through the second polarizer? Because a polarizer does not pass or block whole waves — it transmits the *vector component* of $\vec{E}$ along its axis and absorbs the perpendicular component. Any $\vec{E}$ in the transverse plane can be decomposed along these two axes, so something is always transmitted unless $\vec{E}$ is exactly perpendicular to the axis.

Let the incident beam be plane-polarized along $\hat{y}$,

$$\vec{E}_{\text{inc}}(x,t) \;=\; E_0 \sin(kx-\omega t)\,\hat{y}, \tag{8}$$

and let the second polarizer have transmission axis $\hat{n}$ rotated by angle $\theta$ from $\hat{y}$ in the transverse $y$–$z$ plane. Introduce the perpendicular unit vector $\hat{n}_\perp$ so that $\{\hat{n}, \hat{n}_\perp\}$ is an orthonormal basis for the transverse plane. Decomposing $\hat{y}$ in this basis,

$$\hat{y} \;=\; \cos\theta\,\hat{n} \;+\; \sin\theta\,\hat{n}_\perp, \tag{9}$$

the incident field becomes

$$\vec{E}_{\text{inc}}(x,t) \;=\; E_0 \sin(kx-\omega t)\big[\cos\theta\,\hat{n} + \sin\theta\,\hat{n}_\perp\big]. \tag{10}$$

The polarizer transmits the $\hat{n}$-component and absorbs the $\hat{n}_\perp$-component:

$$\vec{E}_{\text{trans}}(x,t) \;=\; E_0 \cos\theta\,\sin(kx-\omega t)\,\hat{n}. \tag{11}$$

The transmitted wave is plane-polarized along $\hat{n}$ with reduced amplitude $E_0\cos\theta$.

Since intensity is proportional to the square of the electric-field amplitude ($I = \frac{1}{2}c\varepsilon_0 E_0^2$), the transmitted intensity is

$$\boxed{I_2 \;=\; I_1 \cos^2\theta.} \tag{12}$$

This is **Malus's law**. The $\cos^2\theta$ factor has a clear physical interpretation: $\cos\theta$ is the projection of $\vec{E}$ onto the polarizer's transmission axis, and squaring converts the field-amplitude reduction into an intensity reduction.

Two limiting cases are worth noting:

- $\theta = 0$: the two polarizers are aligned. $I_2 = I_1$ (full transmission).
- $\theta = 90°$: the polarizers are "crossed." $I_2 = 0$ (complete absorption).

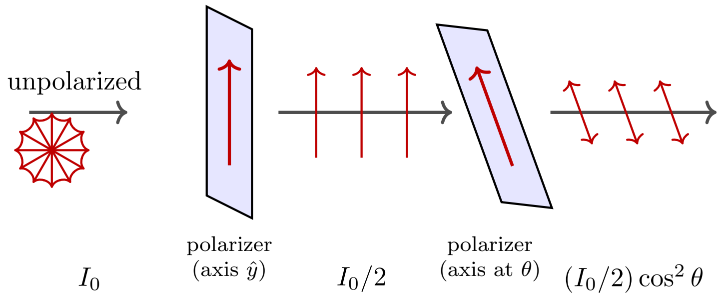


Figure 3: Unpolarized light of intensity $I_0$ is incident from the left. The first polarizer transmits only the component along its axis (here $\hat{y}$), reducing the intensity by half and producing plane-polarized light. A second polarizer with its axis rotated by an angle $\theta$ relative to the first further reduces the intensity to $(I_0/2)\cos^2\theta$, in accordance with Malus's law.

## 4 Electromagnetic Spectrum

For a traveling wave $\sin(kx - \omega t)$, the **wavelength** $\lambda$ is its spatial period and the **period** $T$ its temporal period,

$$\lambda = \frac{2\pi}{k}, \qquad T = \frac{1}{f} = \frac{2\pi}{\omega}. \tag{13}$$

Substituting $k = 2\pi/\lambda$ and $\omega = 2\pi f$ into $c = \omega/k$ (derived earlier),

$$c = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f. \tag{14}$$

The wave equation contains no preferred wavelength: every $\lambda$ (equivalently every frequency $f = c/\lambda$) is an allowed solution. The named bands differ only in how the radiation is produced and detected — the underlying physics is identical. The full spectrum, from short to long wavelength:

| | |
|---|---|
| $\gamma$-rays | $\lesssim 10^{-11}$ m |
| x-rays | $\sim 10^{-11}$–$10^{-8}$ m |
| ultraviolet | $\sim$ 10–400 nm |
| visible | 400–700 nm |
| infrared | 700 nm–1 mm |
| microwave | $\sim$ 1 mm–1 m |
| TV, FM | $\sim$ 1–10 m |
| AM radio | $\sim 10^2$–$10^3$ m |

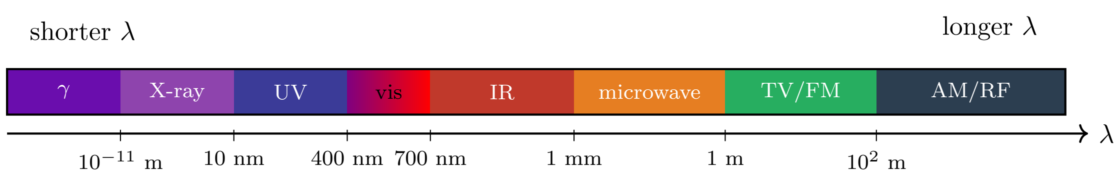


Figure 4: Electromagnetic spectrum spanning $\gamma$-rays through AM/RF, with approximate boundary wavelengths marked on the axis. There are no restrictions on the value of $\lambda$.

The visible band occupies only a narrow portion of the spectrum, located near the peak of the Sun's emission and within a transparent window of the Earth's atmosphere. The other bands are just as physical: radio for communication, microwaves for heating and radar, infrared for thermal imaging, X-rays for medical and structural imaging. All propagate in vacuum at the same speed $c$.