Reflection and Refraction of Light

PHYS 2203 — Introductory Physics III

August 31, 2026

Overview

The previous lectures described light propagating in vacuum. This lecture turns to how light behaves in matter. We first find how a dielectric medium slows a wave and shortens its wavelength, defining the index of refraction. We then study what happens when light meets a boundary between two media: the laws of reflection and refraction, how much of the light is reflected, and total internal reflection.

1 Electromagnetic Waves in a Linear Dielectric Medium

The vacuum analysis must be modified when the wave propagates through a material. A linear dielectric consists of a collection of microscopic electric dipoles. In the absence of an applied field these dipoles are randomly oriented, and the net polarization vanishes. An applied electric field partially aligns the dipoles, producing an internal field opposite to the applied field. The total field within the medium is therefore reduced.

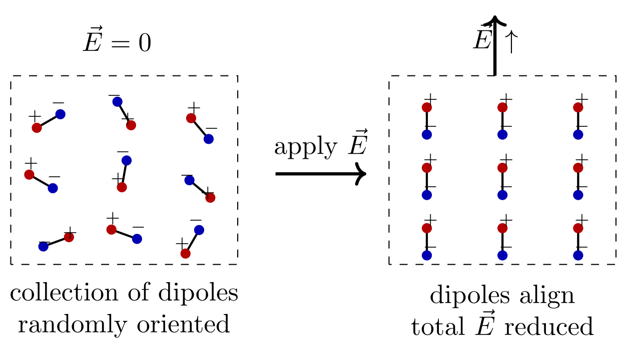

Figure 1: Schematic of the polarization response of a dielectric. With no applied field (left) the molecular dipoles are randomly oriented and average to zero. An applied field $\vec{E}$ (right) partially aligns the dipoles with the field, producing an internal field that partially cancels the applied field. The net field inside the material is reduced.

The macroscopic effect of the polarization is captured by replacing the vacuum permittivity ε_0 with the permittivity of the medium,

$$\varepsilon = \kappa\,\varepsilon_0, \tag{1}$$

where κ is the (dimensionless) *dielectric constant* of the material. In the wave equation, the speed of propagation $c = 1/\sqrt{\mu_0 \varepsilon_0}$ is then replaced by

$$v = \frac{1}{\sqrt{\mu_0\,\varepsilon}} = \frac{c}{\sqrt{\kappa}} = \frac{c}{n}, \tag{2}$$

where

$$\boxed{n = \sqrt{\kappa}} \tag{3}$$

is the **index of refraction** of the medium. For transparent materials $\kappa \geq 1$, so $n \geq 1$ and the speed of light in matter is less than its vacuum value.

medium	n (for visible light)
vacuum (no medium)	1
air	1.00029
water	1.33
glass	1.5
semiconductors	~ 4

Frequency vs. wavelength in a medium

When light enters a new medium its frequency does not change, only its wavelength. Consider the electric field just to the left and right of a boundary at $x = 0$, written as traveling waves,

$$E_L = E_0 \sin(kx - \omega t), \qquad E_R = E_0' \sin\big(k'x - \omega' t\big). \tag{4}$$

Continuity of the field at the boundary requires $E_L = E_R$ at $x = 0$ for all t,

$$E_0 \sin(-\omega t) = E_0' \sin\big(-\omega' t\big) \text{ for all } t \quad \Longrightarrow \quad \omega = \omega'. \tag{5}$$

The frequency is therefore the same on both sides. Because this continuity holds only at the single point $x = 0$, we cannot fix t and scan over x to force $k = k'$. The wavenumber is set instead by the wave speed in each medium: with $v = c/n$ and f fixed, $v = f\lambda$ gives

$$\lambda = \frac{v}{f} = \frac{c}{n\,f} = \frac{\lambda_{\text{vacuum}}}{n}, \tag{6}$$

so the wavelength is *shorter* in a medium with $n > 1$. This change of speed at an interface is what bends light rays, as we now examine.

2 Reflection and Refraction at a Plane Boundary

Light travels in straight lines until it encounters a boundary or aperture. Depending on the size of the optical features relative to the wavelength, three successively more refined descriptions are used.

1. **Geometric optics** (Ch. 32). Light is treated as a collection of rays that travel in straight lines and refract or reflect at boundaries. Interference and diffraction effects are neglected. This description is appropriate when all relevant length scales are much larger than the wavelength.

2. **Physical optics** (Ch. 33). Light is treated as a wave. Consequences of the finite wavelength such as interference and diffraction are explicitly included. This description is required for thin films, gratings, and small apertures.

3. **Quantum optics** (Ch. 3). The discrete nature of light is included: rays, interference, and the *photon* concept all play roles. This description is required when individual photon energies become comparable to the energies involved in the detection process.

We begin with geometric optics, in which light is described by **rays**. A ray is a line pointing along the direction the wave travels, perpendicular to the wavefronts. A narrow beam of light, such as a laser pointer, is a good physical picture of a single ray.

When a light ray reaches a boundary between two transparent media, part of the wave is reflected back into the first medium and part is transmitted (refracted) into the second medium. The directions of the reflected and refracted rays are determined by two empirical laws.

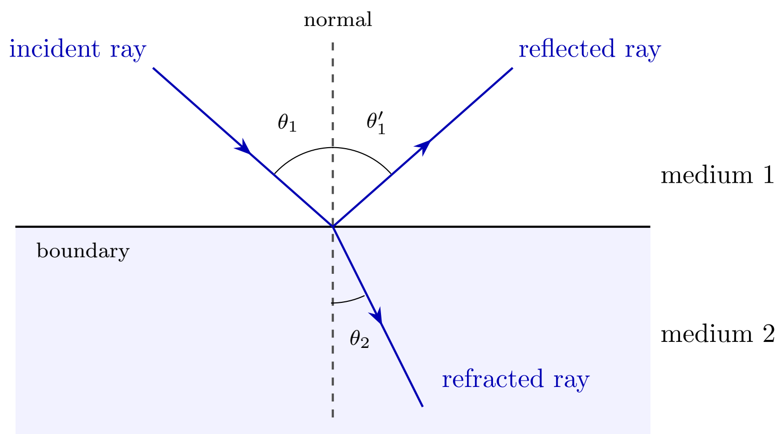

Figure 2: An incident ray strikes the boundary between two media. The reflected ray returns into medium 1 and the refracted ray enters medium 2. All angles are measured from the local normal to the surface.

Law of Reflection

1. The incident ray, the reflected ray, and the normal line at the point of incidence all lie in a single plane, called the **plane of incidence**.

2. The angle of incidence equals the angle of reflection:

$$\boxed{\theta_1 \;=\; \theta'_1} \tag{7}$$

Proof of the law of reflection from Fermat's principle

The law of reflection follows from **Fermat's principle**: between two fixed points, light travels the path that takes the least time. Within a single medium the speed is constant, so least time means least path length.

Light leaves a point A, reflects off a plane mirror at a point P, and arrives at a point B. Place the mirror along the x-axis, with $A = (x_A, y_A)$, $B = (x_B, y_B)$, and the reflection point $P = (x, 0)$ free to slide along the mirror. The total path length is

$$L(x) \;=\; \underbrace{\sqrt{(x - x_A)^2 + y_A^2}}_{AP} \;+\; \underbrace{\sqrt{(x_B - x)^2 + y_B^2}}_{PB}. \tag{8}$$

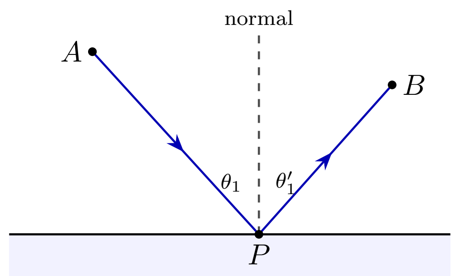

Figure 3: Reflection path from A to B by way of the point P on the mirror. Fermat's principle fixes P by making the total path length $AP + PB$ stationary, which gives $\theta_1 = \theta_1'$.

Fermat's principle requires the path length to be stationary, $dL/dx = 0$:

$$\frac{x - x_A}{\sqrt{(x - x_A)^2 + y_A^2}} - \frac{x_B - x}{\sqrt{(x_B - x)^2 + y_B^2}} = 0. \tag{9}$$

Each fraction is the sine of the angle the corresponding ray makes with the normal to the mirror: the first is $\sin\theta_1$ for the incident ray AP, and the second is $\sin\theta_1'$ for the reflected ray PB. The condition therefore reads $\sin\theta_1 = \sin\theta_1'$, which is the law of reflection, $\theta_1 = \theta_1'$.

Snell's Law of Refraction

1. The refracted ray lies in the plane of incidence.
2. The angles of incidence and refraction satisfy

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2,} \tag{10}$$

   where n_1 and n_2 are the indices of refraction of media 1 and 2, respectively.

Snell's law can also be derived from Fermat's principle, but we will leave this derivation for a homework problem.

3 Intensity of the Reflected Light

The fraction of the incident intensity that is reflected at a boundary depends on three quantities: the angle of incidence θ_1, the mismatch between the indices of refraction n_1 and n_2, and the polarization of the wave. The full angular dependence (the Fresnel equations) is left to a later course. The special case of normal incidence is the most useful for many applications.

Normal incidence ($\theta_1 = 0°$)

Take the boundary to be the plane $x = 0$, with medium 1 (n_1) in $x < 0$ and medium 2 (n_2) in $x > 0$, and let the wave be normally incident from the left. Medium 1 then carries two waves — the incident wave moving right and the reflected wave moving left — while medium 2 carries only the transmitted wave moving right. With $\vec{E}$ along $\hat{y}$,

$$E_1(x,t) = E_i \sin(k_1 x - \omega t) + E_r \sin(k_1 x + \omega t), \qquad E_2(x,t) = E_t \sin(k_2 x - \omega t). \tag{11}$$

The magnetic fields point along $\hat{z}$ with amplitude $|B| = |E|/v$. For a right-moving wave $\hat{E}\times\hat{B} = +\hat{x}$; for the left-moving reflected wave that sign flips, so

$$B_1(x,t) = \frac{E_i}{v_1}\sin(k_1 x - \omega t) - \frac{E_r}{v_1}\sin(k_1 x + \omega t), \qquad B_2(x,t) = \frac{E_t}{v_2}\sin(k_2 x - \omega t). \tag{12}$$

At $x = 0$ the tangential parts of $\vec{E}$ and $\vec{B}$ are continuous. Using $\sin(-\omega t) = -\sin\omega t$ and matching the coefficient of $\sin\omega t$, the conditions hold for all t only if

$$E_i - E_r = E_t, \qquad \frac{E_i + E_r}{v_1} = \frac{E_t}{v_2} \implies n_1\,(E_i + E_r) = n_2\,E_t, \tag{13}$$

where $v = c/n$. Eliminating E_t,

$$n_1(E_i + E_r) = n_2(E_i - E_r) \qquad \Longrightarrow \qquad \frac{E_r}{E_i} = \frac{n_2 - n_1}{n_1 + n_2}. \tag{14}$$

The intensity of a wave scales as $I \propto nE_0^2$, and the incident and reflected waves travel in the *same* medium, so the factor of n_1 cancels in their ratio, leaving

$$\boxed{\frac{I_{\text{reflected}}}{I_0} = \left(\frac{E_r}{E_i}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2,} \tag{15}$$

where I_0 is the incident intensity. The transmitted amplitude follows from $E_t = E_i - E_r$, which gives $E_t/E_i = 2n_1/(n_1 + n_2)$. The transmitted wave now travels in medium 2, so its intensity carries an extra factor of n_2/n_1 relative to the incident wave, and

$$\boxed{\frac{I_{\text{transmitted}}}{I_0} = \frac{n_2}{n_1}\left(\frac{E_t}{E_i}\right)^2 = \frac{4\,n_1 n_2}{(n_1 + n_2)^2}.} \tag{16}$$

As a check, $I_{\text{reflected}}/I_0 + I_{\text{transmitted}}/I_0 = 1$, so energy is conserved at the boundary.

Example: air–glass interface. Taking $n_1 = 1.0$ and $n_2 = 1.5$,

$$\frac{I_{\text{reflected}}}{I_0} = \left(\frac{1 - 1.5}{1 + 1.5}\right)^2 = 0.04. \tag{17}$$

Four percent of the incident intensity is reflected, and 96% is transmitted into the glass at normal incidence.

Angle dependence

For unpolarized light at an air–glass interface, the reflected fraction is small at normal incidence and increases smoothly with angle, approaching 100% as $\theta_1 \to 90^\circ$ (grazing incidence).

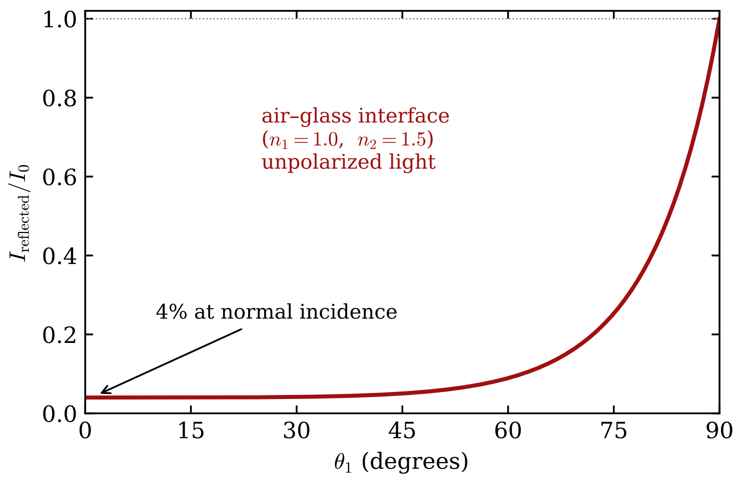

Figure 4: Fraction of incident intensity reflected at an air–glass interface for unpolarized light as a function of incidence angle. At normal incidence ($\theta_1 = 0°$) only $\sim 4\%$ is reflected. At grazing incidence ($\theta_1 \to 90°$) essentially all of the light is reflected. Curve computed from the Fresnel equations.

4 Total Internal Reflection

If medium 2 is a perfect mirror, no refracted ray exists and all of the incident light is reflected. A more interesting situation arises when medium 2 is transparent but the incidence angle in the optically denser medium is large enough that Snell's law admits no real refraction angle. This phenomenon is called **total internal reflection**.

Derivation of the critical angle

Rearranging Snell's law,

$$\sin\theta_2 \;=\; \frac{n_1}{n_2}\sin\theta_1. \tag{18}$$

Because $\sin\theta_2 \le 1$, the equation has no solution when

$$\frac{n_1}{n_2}\sin\theta_1 \;>\; 1. \tag{19}$$

Experiment shows that under these conditions there is no refracted ray. All of the light is reflected back into medium 1. This occurs only when $n_1 > n_2$ (the wave attempts to pass from an optically denser medium to a less dense one).

The boundary between "refraction occurs" and "no refraction occurs" defines the **critical angle** θ_c,

$$\boxed{\sin\theta_c \;=\; \frac{n_2}{n_1} \quad (n_1 > n_2).} \tag{20}$$

Whenever $\theta_1 > \theta_c$, the incident wave undergoes total internal reflection.

Example: water–air interface. For $n_1 = 1.33$ (water) and $n_2 = 1.00$ (air),

$$\theta_c \;=\; \sin^{-1}\!\left(\frac{1.00}{1.33}\right) \;\approx\; 48.8^\circ. \tag{21}$$

A ray traveling upward inside the water at any angle greater than 48.8° from the surface normal cannot escape into the air. It is totally reflected back into the water.

Application: optical fibers and fiber internet

Total internal reflection enables transmission of light over long distances through a transparent core surrounded by a lower-index cladding. For a glass–air interface ($n_1 = 1.5$, $n_2 = 1.0$),

$$\theta_c \;=\; \sin^{-1}\!\left(\frac{1.0}{1.5}\right) \;\approx\; 41.8^\circ. \tag{22}$$

Light injected into the core at angles exceeding θ_c from the surface normal reflects repeatedly off the core–cladding boundary without loss, propagating along the fiber.

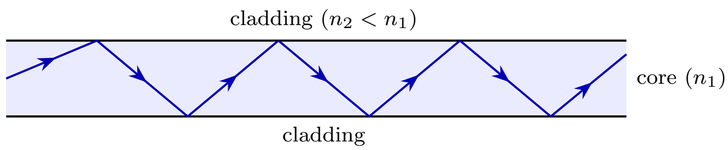

Figure 5: Schematic of an optical fiber. Light injected into the higher-index core strikes the core–cladding boundary at angles above the critical angle and undergoes total internal reflection at each bounce, propagating along the length of the fiber.

This is the physical basis of fiber-optic internet. Data are encoded as pulses of light that travel down glass fibers rather than as electrical signals on copper wire. Because every bounce is a total internal reflection, a light pulse reaches a receiver many kilometers away without leaking out through the walls of the fiber. A metal wire dissipates an electrical signal far more rapidly, which is why long-distance and high-bandwidth links are built from glass fiber.

5 Polarization by Reflection and Brewster's Angle

This section is optional.

The normal-incidence result of Section 3 assumed the electric field lay entirely in the plane of the boundary, so a single amplitude sufficed. When the incident angle is not normal, that derivation must be extended to treat the two polarizations differently: the field splits into a component perpendicular to the plane of incidence and a component lying in it, and applying the boundary conditions to each gives a *different* reflected fraction. These two results are the **Fresnel equations**, whose consequence we simply quote here.

Because the two polarizations reflect with different strengths, unpolarized light becomes *partially* polarized on reflection. At one special angle of incidence — the **polarizing angle**, or **Brewster's angle** θ_p (David Brewster, 1812) — the in-plane component is not reflected at all, and the reflected beam is *completely* polarized with $\vec{E}$ perpendicular to the plane of incidence. At this angle the reflected and refracted rays are perpendicular to one another.

This perpendicularity fixes θ_p. The reflected ray leaves at θ_p from the normal, so for it to be perpendicular to the refracted ray the refraction angle must be $\theta_2 = 90° - \theta_p$. Snell's law then gives

$$n_1 \sin\theta_p \;=\; n_2 \sin\theta_2 \;=\; n_2 \sin(90° - \theta_p) \;=\; n_2 \cos\theta_p, \tag{23}$$

and dividing both sides by $n_1 \cos\theta_p$ gives

$$\boxed{\tan\theta_p \;=\; \frac{n_2}{n_1}.} \tag{24}$$

For light in air striking water, $\tan\theta_p = 1.33$ and $\theta_p \approx 53°$.

Because reflected light is polarized, sunglasses containing a polarizing sheet are very effective at cutting out glare. Light reflected from a horizontal surface — a lake, snow on the ground, a road — has its electric field predominantly *horizontal*, so polarizing sunglasses are made with a *vertical* transmission axis: they absorb most of the reflected glare while merely halving the intensity of the rest of the (unpolarized) scene. If you have polarizing sunglasses, you can observe this by looking at glare through the glasses and rotating them 90° — much more of the reflected light is transmitted.

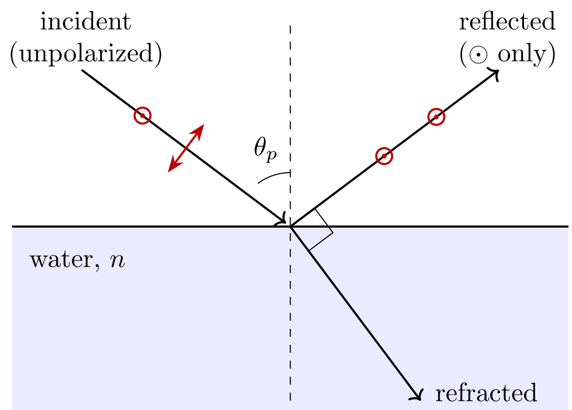

Figure 6: Unpolarized light incident on a horizontal water surface at the polarizing angle θ_p, where the reflected and refracted rays are perpendicular. The incident beam contains both polarizations: $\vec{E}$ in the plane of incidence (red double arrow) and perpendicular to it (⊙, out of the page). Only the perpendicular component appears in the reflected beam, so the reflected light is completely polarized — horizontally, for this horizontal surface.