# Optical Images: Spherical Mirrors

PHYS 2203 — Introductory Physics III

September 2, 2026

## Overview

This lecture develops the formation of *optical images*: when a mirror redirects the rays leaving a point on an object so that they converge to (or appear to diverge from) a common point, an image forms. We treat reflection from plane and **spherical mirrors**, leading to the mirror equation and magnification (Sec. 32.1). The system is analyzed in two independent ways: graphically, with a ray diagram, and algebraically, with an image equation. The next lecture turns to image formation by *refraction* at curved surfaces and builds the thin lens.

## 1 Optical Images: Mirrors

**Image formation.** If light rays emerging from a point on an object are caused (by a mirror or lens) to converge to a common point — or to diverge as if from a common point — an *image* of the object is formed there. We treat spherical mirrors first, then refracting surfaces.

### Spherical mirrors: geometry and definitions

A spherical mirror is a section of a sphere with a reflecting inner (concave) or outer (convex) surface. The center of the sphere is the **center of curvature** $C$; the radius is the **radius of curvature** $r$. The line through $C$ and the mirror's vertex is the **optic axis**. The **focal point** $F$ is located halfway between $C$ and the vertex along the optic axis,

$$f = \frac{r}{2}, \qquad \text{(focal length)} \tag{1}$$

and is where rays parallel to the axis meet, or appear to have come from, after reflection.

### Concave and convex mirrors

A **concave** mirror reflects from its inner surface, so its center of curvature $C$ and focal point $F$ lie *in front of* the mirror. Rays parallel to the axis converge to $F$ after reflection, so a concave mirror *concentrates* light: aimed at the Sun, it funnels the captured beam through $F$, and its focal length is positive, $f = r/2$.

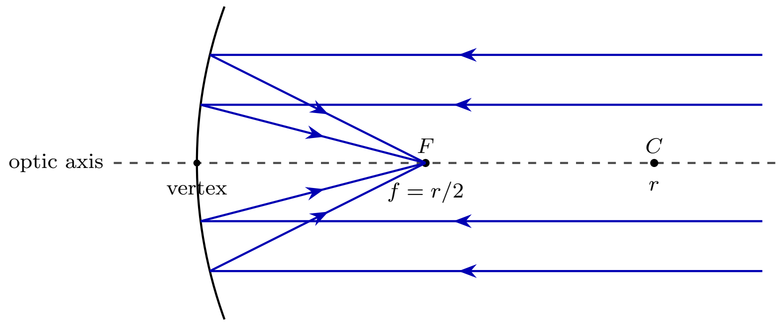


Figure 1: Rays parallel to the optic axis reflect off a concave spherical mirror and converge at the focal point $F$, located halfway between the vertex and the center of curvature $C$.

A **convex** mirror reflects from its outer surface, so its center of curvature $C$ and focal point $F$ lie *behind* the mirror. Rays parallel to the axis now reflect so as to *diverge*, as though they had come from a point behind the mirror. Extending each reflected ray backward locates that *virtual* focal point $F$, a distance $r/2$ behind the vertex. A convex mirror therefore spreads parallel light instead of concentrating it, and by the sign convention below both its radius of curvature and its focal length are negative: $r < 0$ and $f = r/2 < 0$.

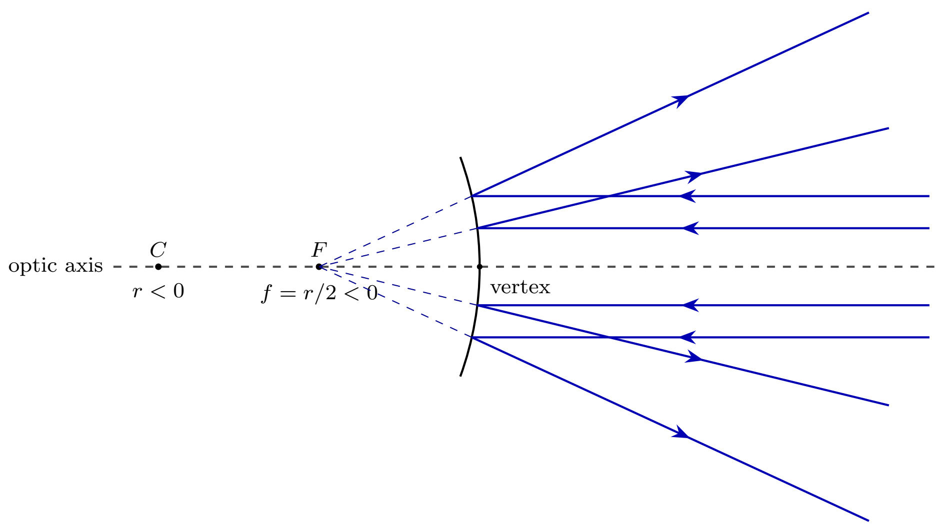


Figure 2: Rays parallel to the optic axis reflect off a convex spherical mirror and *diverge*. Their backward extensions (dashed) meet at a *virtual* focal point $F$, a distance $|r|/2$ behind the vertex, halfway to the center of curvature $C$. Because $F$ and $C$ lie behind the mirror, the sign convention makes $r$ and $f$ negative. A convex mirror spreads parallel light rather than concentrating it.

### Paraxial-ray approximation

The image formed by a spherical mirror depends on the angles at which the rays strike the surface. A simple description is obtained by restricting to **paraxial rays** — rays that make small angles with the optic axis and that strike the mirror near the vertex. All results below assume paraxial rays.

**Ray diagrams: four principal rays**

Within this approximation, four principal rays from an object point are used:

(a) A ray parallel to the optic axis reflects through the focal point $F$ (the definition of $F$).

(b) A ray passing through $F$ reflects parallel to the optic axis (ray (a) reversed).

(c) A ray passing through the center of curvature $C$ reflects back along itself (every radius is perpendicular to the surface, so it strikes the mirror head-on).

(d) A ray incident on the vertex reflects at an equal angle on the other side of the optic axis (there, the axis *is* the normal).

Any two of these four rays drawn from the same point on the object will intersect (or, when extended backward, appear to intersect) at the corresponding image point; a third serves as a check. If the rays actually cross, the image is **real**; if their backward extensions cross behind the mirror, the image is **virtual**.

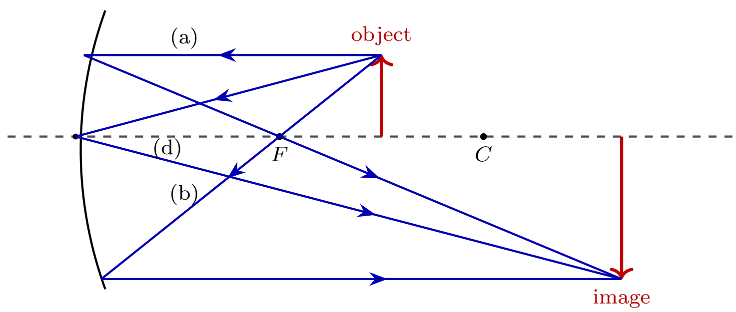


Figure 3: Three principal-ray construction of the image of an object placed between $C$ and $F$ in front of a concave mirror, using the parallel ray (a), the focal ray (b), and the vertex ray (d), which reflects at an equal angle on the far side of the optic axis. The reflected rays intersect at a point beyond $C$, producing a real, inverted, magnified image.

## 2 Mirror Equation and Magnification

The ray diagram contains enough geometry to *derive* the relation among the object distance $S$, the image distance $S'$, and the focal length $f$. Two of the principal rays suffice.

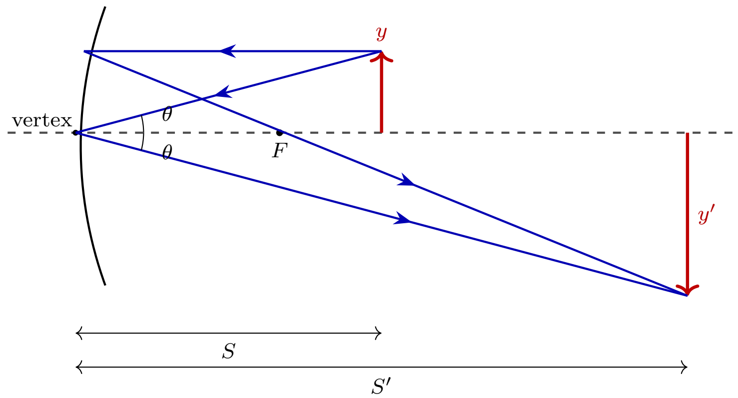


Figure 4: Deriving the mirror equation from the vertex ray (equal angles $\theta$ above and below the axis) and the parallel ray (through $F$ after reflection).

**Step 1 — the vertex ray gives the magnification.** The vertex ray makes equal angles with the axis before and after reflection, so the object and image triangles at the vertex are similar:

$$\tan\theta \;=\; \frac{y}{S} \;=\; \frac{|y'|}{S'}. \tag{2}$$

The image is inverted, so $y' = -|y'|$, and the **transverse magnification** is

$$\boxed{m \;=\; \frac{y'}{y} \;=\; -\frac{S'}{S}.} \tag{3}$$

**Step 2 — the parallel ray gives a second relation.** The parallel ray reflects at height $y$ above the vertex and travels in a straight line through $F$ to the image plane at $S'$, where its height is $y'$ (all rays from the object tip meet at the image tip). This straight line cuts off a triangle on each side of $F$: before $F$, one with legs $y$ (at the mirror) and $f$ (mirror to focal point); after $F$, one with legs $|y'|$ (at the image plane) and $S' - f$ (focal point to image plane). The two triangles share vertical angles at $F$, so they are similar:

$$\frac{y}{f} \;=\; \frac{|y'|}{S'-f}. \tag{4}$$

Since the image tip lies below the axis, $y' = -|y'|$, and solving for $y'$,

$$y' \;=\; -\,y\,\frac{S'-f}{f} \;=\; y\left(1 - \frac{S'}{f}\right). \tag{5}$$

**Step 3 — combine.** Setting the two expressions for $y'/y$ equal,

$$-\frac{S'}{S} \;=\; 1 - \frac{S'}{f} \qquad \xrightarrow{\text{divide by } S'} \qquad -\frac{1}{S} \;=\; \frac{1}{S'} - \frac{1}{f}, \tag{6}$$

which rearranges to

$$\boxed{\frac{1}{S} + \frac{1}{S'} \;=\; \frac{1}{f}} \qquad \text{(spherical mirror equation)} \tag{7}$$

The sign of $m$ encodes orientation: $m > 0$ means an upright image (same orientation as the object), $m < 0$ means an inverted image. The magnitude $|m|$ gives the linear size ratio: $|m| < 1$ for a reduced image, $|m| > 1$ for a magnified image.

### Sign conventions

The derivation assumed a concave mirror forming a real, inverted image. The same equation covers convex mirrors with virtual images, provided signed distances are used according to the following conventions:

- $S > 0$ when the object is in front of the mirror.
- $S' > 0$ when the image is in front of the mirror (*real image*); $S' < 0$ when the image is behind the mirror (*virtual image*).
- $r > 0$ (and hence $f > 0$) for a concave mirror.
- $r < 0$ (and hence $f < 0$) for a convex mirror.
- $r = \infty$ (and hence $f = \infty$) for a plane mirror.

With both tools now in hand, it is good practice to locate every image both graphically and algebraically, letting each method serve as a check on the other. The worked examples below exercise these rules on all three mirror types.

## 3 Worked Examples: Spherical Mirrors

### Example 1: the plane mirror as a limiting case

A flat mirror is the $r \to \infty$ limit of a spherical mirror: $f = r/2 \to \infty$, and the mirror equation gives

$$\frac{1}{S} + \frac{1}{S'} = 0 \quad \Longrightarrow \quad S' = -S, \qquad m = -\frac{S'}{S} = +1.$$

The image is as far behind the mirror as the object is in front, virtual ($S' < 0$), upright, and the same size as the object. The ray construction confirms this: after reflection, the rays diverge exactly as if they originated at the image point behind the mirror, where no light actually goes.

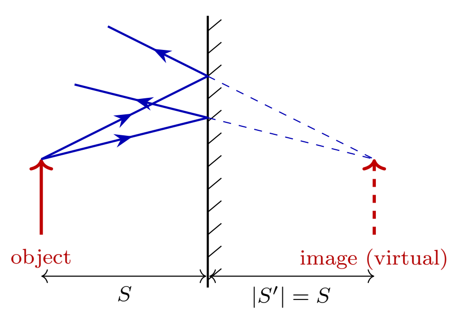


Figure 5: A plane mirror. Reflected rays (solid) diverge as if they originated at the image point behind the mirror (dashed extensions), at a distance equal to the object distance. The image is virtual, upright, and the same size as the object.

### Example 2: hummingbird in a convex garden mirror

A hummingbird is located 20 cm in front of a convex spherical garden mirror with radius of curvature 10 cm.

**Given.**

$$\begin{aligned} S &= 20 \text{ cm} \\ r &= -10 \text{ cm} \quad (\text{convex, so } r < 0) \\ f &= \frac{r}{2} = -5 \text{ cm.} \end{aligned}$$

**Image distance.** Solving the mirror equation,

$$\frac{1}{S'} \;=\; \frac{1}{f} - \frac{1}{S} \;=\; -\frac{1}{5 \text{ cm}} - \frac{1}{20 \text{ cm}} \;=\; -\frac{4}{20 \text{ cm}} - \frac{1}{20 \text{ cm}} \;=\; -\frac{5}{20 \text{ cm}} \;=\; -\frac{1}{4 \text{ cm}},$$

so

$$S' \;=\; -4 \text{ cm.}$$

The negative sign indicates a virtual image (behind the mirror).

**Magnification.**

$$m \;=\; -\frac{S'}{S} \;=\; -\frac{-4}{20} \;=\; +\frac{1}{5} \;=\; +0.2.$$

The image is **upright** ($m > 0$) and **reduced** in size ($|m| < 1$). This is the familiar behavior of a convex security or surveillance mirror, which provides a wide-field-of-view image that is upright but smaller than the object.

### Example 3: concave spoon (general behavior)

For a concave mirror, $r > 0$ and $f > 0$. The magnification as a function of object distance is obtained by combining the magnification formula with the mirror equation:

$$\begin{aligned} m \;&=\; -\frac{S'}{S} \;=\; -\frac{1}{S}\left(\frac{1}{f} - \frac{1}{S}\right)^{-1} \\ &=\; -\frac{1}{S} \cdot \frac{fS}{S-f} \;=\; -\frac{f}{S-f} \;=\; \frac{f}{f-S}. \end{aligned} \tag{8}$$

**Behavior with respect to object distance.**

- $m < 0$ (inverted) when $S > f$.
- $m > 0$ (upright) when $S < f$.
- $|m| \to \infty$ as $S \to f$.

The transition between upright and inverted images, with divergent magnification, occurs as the object passes through the focal point.

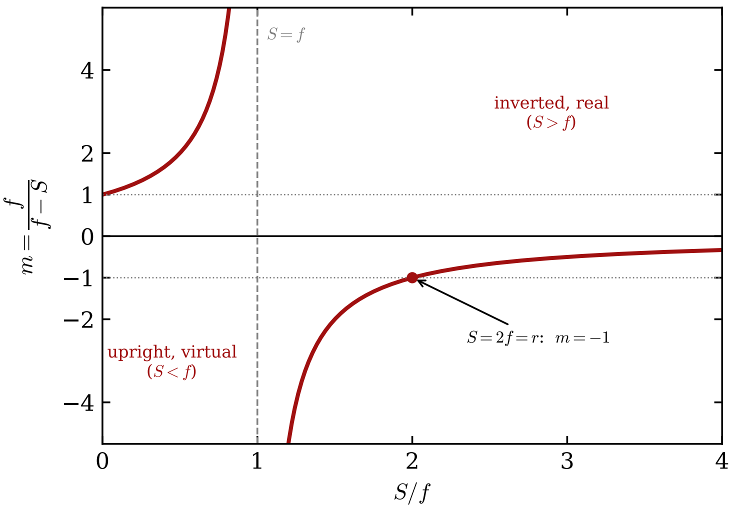


Figure 6: Magnification of a concave mirror as a function of object distance. For $S < f$, $m > 0$ (upright virtual image). For $S > f$, $m < 0$ (inverted real image). The magnification diverges at $S = f$ (vertical asymptote); an object at the center of curvature ($S = 2f = r$) is imaged at the same size, inverted ($m = -1$).